Basic operation questions for CDSE, CGL Tier 2, CGLTier 1 and SSC 10+2

Basic operation quiz 5

Direction: Study the following questions carefully and choose the right answer.

1. When $2x + \frac{2}{x} = 3$, then value of $x^3 + \frac{1}{x^3} + 2$ is

A. 2/7 B. 7/8

C. 7/2 D. 8/7

2. If x = 332, y = 333, z = 335 then the value of $x^3 + y^3 + z^3 - 3xyz$ = ?

A. 1000 B. 9000

C. 7000 D. 65000

3. If $x + \frac{1}{x} = 5$, then $\frac{2x}{3x^2 - 5x + 3}$ is equal to

A. 5 B. 1/5

C. 3 D. 1/3

4. If $ab + bc + ca = 0$, then the value of $\frac{1}{a^2 - bc} + \frac{1}{b^2 - ac} + \frac{1}{c^2 - ab}$ is

A. 2 B. –1

C. 0 D. 1

5. If x = 2 then the value of $x^3 + 27x^2 + 243x + 631$ is:

A. 1211 B. 1231

C. 1233 D. 1321

6. When $2x + \frac{2}{x} = 3$, then value of $x3 + \frac{1}{x^3} + 2$ is

A. 2/7 B. 7/8

C. 7/2 D. 8/7

7. If $x^2 + 1x^2 = 66$, then the value of $\frac{x^2 - 1 + 2x}{x} = ?$

A. ± 8 B. 10 , –6

C. 6, –10 D. ± 4

8. If $x^2 - 3x + 1 = 0$, then the value of $(x^6 + x^4 + x^2 + 1)x^3$ will be

A. 18 B. 15

C. 21 D. 30

9. Given, $x^3 + \frac{3}{x} = 4(a^3 + b^3)$ and $3^x + \frac{1}{x^3} = 4(a^3 - b^3)$ then $a^2 - b^2$ is equal to

A. 4 B. 0

C. 1 D. 2

10. If $x^2 + y^2 + 1 = 2x$, then the value of $x^5 + y^7$ is

A. 2 B. 0

C. –1 D. 1

Correct answers:

1	2	3	4	5	6	7	8	9	10
B	C	B	C	C	B	B	C	C	D

Explanations:

1). Given,

$$2x + \frac{2}{x} = 3$$

$$\text{or } (x + \frac{1}{x}) = \frac{3}{2}$$

Cubing both sides, we get

$$x^3 + \frac{1}{x^3} + 3(x + \frac{1}{x}) = \frac{27}{8}$$

$$\text{or, } x^3 + \frac{1}{x^3} + 3\times \frac{3}{2} = \frac{27}{8}$$

$$\text{or, } x^3 + \frac{1}{x^3} = \frac{27}{8} - \frac{9}{2} = -\frac{9}{8}$$

$$\text{or, } x^3 + \frac{1}{x^3} + 2 = 2 - \frac{9}{8} \Rightarrow \frac{7}{8}$$

Hence, option B is correct.

2). $\because \quad (x^3 + y^3 + z^3 - 3xyz) = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$$= (x + y + z)\times\frac{1}{2}[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

On putting the values, we get

$= 1000 \times \frac{1}{2}[(332 - 333)^2 + (333 - 335)^2 + (335 - 332)^2$

$= 1000 \times \frac{1}{2}[1 + 4 + 9]$

$= 7000$

Hence, option C is correct.

3). $x + \frac{1}{x} = 5$

$\Rightarrow x^2 - 5x + 1 = 0$

Equation multiplied by 3, then

$\Rightarrow 3x^2 - 15x + 3 = 0 \quad \Rightarrow 3x^2 + 3 = 15x$

$\therefore \quad \frac{2x}{3x^2 - 5x + 3} = \frac{2x}{15x - 5x}$

$= \frac{2x}{10x} = \frac{1}{5}$

Hence, option B is correct.

4). $ab + bc + ca = 0$

$\Rightarrow ab + ca = -bc$

$\therefore a^2 - bc = a^2 + ab + ca = a(a + b + c)$

Similarly,

$b^2 - ac = b(a + b + c)$

$c^2 - ac = c(a + b + c)$

$\therefore \quad \frac{1}{a^2 - bc} + \frac{1}{b^2 - ac} + \frac{1}{c^2 - ab}$

$= \quad \frac{1}{a(a + b + c)} + \frac{1}{b(a + b + c)} + \frac{1}{c(a + b + c)}$

$= \quad \frac{bc + ac + ab}{abc(a + b + c)} = 0$

Hence, option C is correct.

5). Given equation,

$f(x) = x^3 + 27x^2 + 243x + 631$

$\Rightarrow x(x^2 + 27x + 243) + 631$

Now, put the value of x = 2

$\Rightarrow 2(2^2 + 27 \times 2 + 243) + 631$

$\Rightarrow 2 (4 + 54 + 243) + 631$

$\Rightarrow 2(301) + 631 = 602 + 631 = 1233.$

6). Given,

$2x + \frac{2}{x} = 3$

or $(x + \frac{1}{x}) = \frac{3}{2}$

Cubing both sides, we get

$x^3 + \frac{1}{x^3} + 3(x + \frac{1}{x}) = \frac{27}{8}$

or, $x^3 + \frac{1}{x^3} + 3 \times \frac{3}{2} = \frac{27}{8}$

or, $x^3 + \frac{1}{x^3} = \frac{27}{8} - \frac{9}{2} = -\frac{9}{8}$

or, $x^3 + \frac{1}{x^3} + 2 = 2 - \frac{9}{8} \Rightarrow \frac{7}{8}$

Hence, option B is correct.

7). $x^2 + \frac{1}{x^2} = 66$

$= (x - \frac{1}{x})^2 + 2 = 66$

$= (x - \frac{1}{x})^2 = 66 - 2 = 64$

$= (x - \frac{1}{x}) = \pm 8$

Now, $\frac{x^2 - 1 + 2x}{x} = \frac{x^2}{x} - \frac{1}{x} + \frac{2x}{x}$

$= x - \frac{1}{x} + 2$ **....(i)**

Putting the value of $(x - \frac{1}{x})$ in eqn (i)

we get, = 8 + 2 or –8 + 2 = 10 or –6.

Hence, option B is correct.

8). Given,

$x^2 – 3x + 1 = 0$

$\Rightarrow x^2 + 1 = 3x \Rightarrow \frac{x^2 + 1}{x} = 3$

$\Rightarrow (x + \frac{1}{x}) = 3$ **....(i)**

Cubing both sides here, we get

$x^3 + \frac{1}{x^3} + 3 (x + \frac{1}{x}) = 27$

$\Rightarrow (x^3 + \frac{1}{x^3}) + 3 \times 3 = 27$

$\Rightarrow (x^3 + \frac{1}{x^3}) = 27 – 9 = 18$ **...(ii)**

Now, $\frac{x^6 + x^4 + x^2 + 1}{x^3} = \frac{x^6}{x^3} + \frac{x^4}{x^3} + \frac{x^2}{x^3} + \frac{1}{x^3}$

$= x^3 + x + \frac{1}{x} + \frac{1}{x^3}$

$= (x^3 + \frac{1}{x^3}) + (x + \frac{1}{x})$

From eqn. (i) and (ii), we get

$= 18 + 3 = 21$

Hence, option C is correct.

9). $x^3 + \frac{3}{x} = 4(a^3 + b^3)$ ….(i)

$3x + \frac{1}{x^3} = 4(a^3 - b^3)$ **.....(ii)**

On adding both equations, we get

$x^3 + 3x + \frac{3}{x} + \frac{1}{x^3} = 8a^3$

$\Rightarrow (x + \frac{1}{x})^3 = (2a)^3$

$\Rightarrow x + \frac{1}{x} = 2a \Rightarrow a = \frac{1}{2}(x + \frac{1}{x})$

Similarly, on subtracting

$x^3 + \frac{3}{x} - 3x - \frac{1}{x^3} = 8b^3$

$\Rightarrow (x - \frac{1}{x})^3 = (2b)^3$

$\Rightarrow b = \frac{1}{2}(x - \frac{1}{x})$

$\therefore a^2 - b^2$

$= \frac{1}{4}[(x + \frac{1}{x})^2 - (x - \frac{1}{x})^2]$

$= \frac{1}{4}[(x + \frac{1}{x} + x - \frac{1}{x})(x + \frac{1}{x} - x + \frac{1}{x})]$

$= \frac{1}{4}[2x \times \frac{2}{x}]$

$= \frac{1}{4} \times 4 = 1.$

Hence, option C is correct.

10). $x^2 + y^2 + 1 = 2x$

$\Rightarrow x^2 + y^2 + 1 - 2x = 0$

$\Rightarrow x^2 - 2x + 1 + y^2 = 0$

$\Rightarrow (x - 1)^2 + y^2 = 0$

In the above eq. the L.H.S. can only become zero when the base of terms; $(x - 1)$ and y become zero because for any other value the sum of their squares will always be a positive integer.

Taking $(x - 1) = 0$

and $y = 0$

Therefore, $x = 1$ and $y = 0$

$\therefore \quad x^5 + y^7 = 1 + 0 = 1.$