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6. If $x = \frac{1}{\sqrt{5}-2}$ then $x^4 + 16x^2 - 8x^3 = ?$

A. -5

B. 1

C. $-2\sqrt{5}$

D. $\sqrt{5} + 2$

7. If $x = \frac{1}{\sqrt{5}-2}$ then $2x^3 - 5x^2 - 14x - 3 = ?$

A. 0

B. 1

C. 5

D. 3

8. If $x + \frac{1}{x} = 1$ then $x^{50} + x^{51} + x^{52} + x^{53} + x^{54} + x^{55} = ?$

A. 6

B. 3

C. 2

D. 0

9. If $\sqrt{x} + \frac{1}{\sqrt{x}} = 1$ then $x^{512} + \frac{1}{x^{512}} = ?$

A. 1

B. -1

C. 2

D. 0

10. If $x + \frac{1}{x} = 0$ then $x^{12} + x^{14} + x^{16} + x^{18} = ?$

A. 4

B. 1

C. 2

D. 0

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Correct answers:

1	2	3	4	5	6	7	8	9	10
D	C	A	A	C	B	A	D	B	D

Explanations:

1). Let $x^2 - y^2 = a$

$$1 - x^2 = b$$

$$y - 1 = c$$

$$a + b + c = 0$$

$$a + b + c = 3abc$$

$$\therefore \frac{(x^2 - y^2)^2}{(1 - x)^2 (y^2 - 1)} + \frac{(1 - x^2)^2}{(x^2 - y^2) (y^2 - 1)} + \frac{(y^2 - 1)^2}{(x^2 - y^2) (y^2 - 1)}$$

$$= \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$$

$$= \frac{a^3 + b^3 + c^3}{abc}$$

$$= \frac{3abc}{abc}$$

$$= 3$$

Hence, option D is correct.

2). $x = \sqrt{3} + \sqrt{4} + \sqrt{5}$

$$x - 2 = \sqrt{3} + \sqrt{5}$$

$$x^2 - 4x + 4 = 8 + 2\sqrt{15}$$

$$x^2 - 4x - 4 = 2\sqrt{15}$$

$$x^4 + 16x^2 + 16 - 8x^3 + 32x - 8x^2 = 60$$

$$x^4 - 8x^3 + 8x^2 + 32x = 44$$

Hence, option C is correct.

$$3). \quad \frac{a^{28} + 1}{a^{14}} = 23$$

$$a^{14} + \frac{1}{a^{14}} = 23$$

$$a^{14} + \frac{1}{a^{14}} + 2 = 25$$

$$a^7 + \frac{1}{a^7} = 5$$

$$a^{21} + \frac{1}{a^{21}} = 5^3 - 3 \times 5$$

$$\frac{a^{42} + 1}{a^{21}} = 110$$

Hence, option A is correct.

$$4). \quad \frac{a^2 + b^2 + c^2}{a^2 - b^2 - c^2} + \frac{b^2 + c^2 + a^2}{b^2 - c^2 - a^2} + \frac{c^2 + a^2 + b^2}{c^2 - a^2 - b^2} - 3 + 3$$

$$= \frac{a^2 + b^2 + c^2 + a^2 - b^2 - c^2}{a^2 - b^2 - c^2} + \frac{b^2 + c^2 + a^2 + b^2 - c^2 - a^2}{b^2 - c^2 - a^2} + \frac{c^2 + a^2 + b^2 + c^2 - a^2 - b^2}{c^2 - a^2 - b^2} - 3$$



$$= \frac{2a^2}{a^2 - b^2 - c^2} + \frac{2b^2}{b^2 - c^2 - a^2} + \frac{2c^2}{c^2 - a^2 - b^2} - 3$$

Now, $a + b = c$

$$a = c - b$$

$$a^2 - b^2 - c^2 = -2bc \dots\dots\dots(i)$$

and $a + b = c$

$$b = c - a$$

$$b^2 - c^2 - a^2 = -2ac \dots\dots\dots(ii)$$

and $a + b = c$

$$c^2 - a^2 - b^2 = 2ab \dots\dots\dots(iii)$$

$$a + b = c$$

$$a + b - c = 0$$

$$a^3 + b^3 - c^3 = -3abc \dots\dots\dots(iv)$$

$$-a^3 - b^3 + c^3 = 3abc$$

$$= \frac{2a^2}{-2bc} + \frac{2b^2}{-2ac} + \frac{2c^2}{2ab} - 3$$

$$= \frac{-a^2}{bc} - \frac{b^2}{ac} + \frac{c^2}{ab} - 3$$

$$= \frac{-a^3 - b^3 + c^3}{abc} - 3$$



$$= \frac{3abc}{abc} - 3$$

$$= 0$$

Hence, option A is correct.

5). $x^2 + \frac{1}{x^2} = 7$

$$x + \frac{1}{x} = 3$$

$$x - \frac{1}{x} = \sqrt{5}$$

$$2x = 3 + \sqrt{5}$$

$$x = \frac{3 + \sqrt{5}}{2}$$

$$x = \frac{6 + 2\sqrt{5}}{4}$$

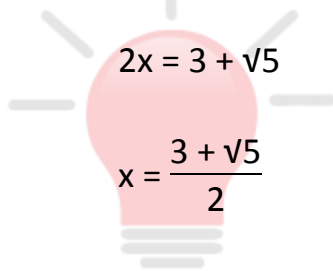
$$x = \left(\frac{\sqrt{5} + 1}{2}\right)^2$$

$$\sqrt{x} = \frac{\sqrt{5} + 1}{2}$$

$$\sqrt{x}(\sqrt{x} - 1) = \left(\frac{\sqrt{5} + 1}{2}\right)\left(\frac{\sqrt{5} + 1}{2} - 1\right)$$

$$= \frac{\sqrt{5} + 1}{2} \times \frac{\sqrt{5} - 1}{2} = \frac{5 - 1}{4} = 1$$

Hence, option C is correct.



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$$6). \quad x = \frac{1}{\sqrt{5} - 2}$$

$$x = \sqrt{5} + 2$$

$$x - 2 = \sqrt{5}$$

$$x^2 - 4x + 4 = 5$$

$$x^2 - 4x - 1 = 0 \dots\dots\dots(i)$$

$$x^2 - 4x + 1 = 2 \dots\dots\dots(ii)$$

Multiply eq. (i) by eq. (ii)

$$(x^2 - 4x - 1)(x^2 - 4x + 1) = 0$$

$$(x^2 - 4x)^2 - 1 = 0$$

$$x^4 + 16x^2 - 8x^3 = 1$$

Hence, option B is correct.

$$7). \quad x = \frac{1}{\sqrt{5} - 2}$$

$$x = \sqrt{5} + 2$$

$$x - 2 = \sqrt{5}$$

$$x^2 - 4x + 4 = 5$$

$$x^2 - 4x - 1 = 0$$

$$\begin{array}{r}
 x^2 - 4x - 1 \overline{) 2x^3 - 5x^2 - 14x - 3} \quad (2x + 3) \\
 \underline{2x^3 - 8x^2 - 2x} \\
 + 3x - 12x - 3 \\
 - 9x - 3 \\
 - 3 \\
 \quad 0
 \end{array}$$

$$\therefore 2x^3 - 5x^2 - 14x - 3 = (x^2 - 4x - 1)(2x + 3)$$

$$= 0$$

Hence, option A is correct.

$$8). \quad x + \frac{1}{x} = 1$$

$$x^2 - x + 1 = 0$$

$$x^3 + 1 = 0$$

$$x^3 = -1$$

$$x^{50} + x^{51} + x^{52} + x^{53} + x^{54} + x^{55}$$

$$= x^{50} + x^{51} + x^{52} + x^{53} + x^{54} + x^{55}$$

$$= x^{50} + x^3 \cdot x^{50} + x^{51} + x^3 \cdot x^{51} + x^{52} + x^3 \cdot x^{52}$$

$$= x^{50} - x^{50} + x^{51} - x^{51} + x^{52} - x^{52}$$

$$= 0$$

Hence, option D is correct.

$$9). \quad \sqrt{x} + \frac{1}{\sqrt{x}} = 1$$

$$x + \frac{1}{x} + 2 = 1$$

$$x + \frac{1}{x} = -1$$

$$x^2 + \frac{1}{x^2} + 2 = 1$$

$$x^2 + \frac{1}{x^2} = -1$$

$$x^4 + \frac{1}{x^4} = -1$$

$$x^8 + \frac{1}{x^8} = -1$$

Similarly

$$x^{512} + \frac{1}{x^{512}} = -1$$

Hence, option B is correct.

10). $x + \frac{1}{x} = 0$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$x^4 = 1$$

$$x^{12} + x^{14} + x^{16} + x^{18} = (x^4)^3 + (x^2)^7 + (x^4)^4 + (x^2)^9$$

$$= 1 - 1 + 1 - 1$$

$$= 0$$

Hence, option D is correct.

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