Polynomials questions for CDSE

Polynomials quiz 1

Directions: Study the following questions carefully and choose the right answer:

1. What is the LCM of $a^3b - ab^3$, $a^3b^2 + a^2b^3$ and $ab(a + b)$?

A. $a^2b^2(a^2 - b^2)$ B. $ab(a^2 - b^2)$

C. $a^2b^2 + ab^3$ D. $a^3b^3(a^2 - b^2)$

2. What is the HCF of the polynomials $x^4 - 3x + 2$, $x^3 - 3x^2 + 3x - 1$ and $x^4 - 1$?

A. $x - 1$ B. $x + 1$

C. $x^2 - 1$ D. None of these

3. If $(x - 6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$, then what is the value of k?

A. 3 B. 5

C. 6 D. 8

4. The HCF of $(x^4 - y^4)$ and $(x^6 - y^6)$ is

A. $x^2 - y^2$ B. $x - y$

C. $x^3 - y^3$ D. $x^4 - y^4$

5. What is the LCM of $x^2 + 2x - 8$, $x^3 - 4x^2 + 4x$ and $x^2 + 4x$?

A. $x(x + 4)(x - 2)^2$ B. $x(x + 4)(x - 2)$

C. $x(x + 4)(x + 2)^2$ D. $x(x + 4)^2(x - 2)$

6. What is the HCF of $8(x^5 - x^3 + x)$ and $28 (x^6 + 1)$?

A. $4(x^4 - x^2 + 1)$ B. $2(x^4 - x^2 + 1)$

C. $(x^4 - x^2 + 1)$ D. None of these

7. The LCM of $(x^3 - x^2 - 2x)$ and $(x^3 + x^2)$ is

A. $x^3 - x^2 - 2x$ B. $x^2 + x$

C. $x^4 - x^3 - 2x^2$ D. $x - 2$

8. What is the HCF of $a^2b^4 + 2a^2b^2$ and $(ab)^7 - 4a^2b^9$?

A. ab B. a^2b^3

C. a^2b^2 D. a^3b^2

9. If (x + k) is the HCF of $(x^2 + ax + b)$ and $(x^2 + cx + d)$, then what is the value of k?

$A.\frac{b+d}{a+c}$ $B.\frac{b+d}{c+d}$

$C.\frac{a-b}{c-d}$ $d.\frac{b-d}{a-c}$

10. The HCF and LCM of two polynomials are (x + y) and $(3x^5 + 5x^4y + 2x^3y^2 - 3x^2y^3 - 5xy^4 - 2y^5)$ respectively. If one of the polynomials is $(x^2 - y^2)$, then the other polynomial is

A. $3x^4 - 8x^3y + 10x^2y^2 + 7xy^3 - 2y^4$

B. $3x^4 - 8x^3y - 10x^2y^2 + 7xy^3 - 2y^4$

C. $3x^4 + 8x^3y + 10x^2y^2 + 7xy^3 + 2y^4$

D. $3x^4 + 8x^3y - 10x^2y^2 + 7xy^3 + 2y^4$

Correct answers:

1	2	3	4	5	6	7	8	9	10
A	A	B	A	A	A	C	C	D	C

Explanations:

1). Let $f(x) = a^3b - ab^3 = ab(a^2 - b^2) = ab(a - b)(a + b)$

$g(x) = a^3b^2 + a^2b^3 = a^2b^2(a + b)$

and $h(x) = ab(a + b) = ab(a + b)$

$\therefore$ LCM of $[f(x), g(x), h(x)] = a^2b^2(a + b)(a - b) = a^2b^2\ (a^2 - b^2)$

Hence, option A is correct.

2). Let $f(x) = x^4 - 3x + 2 = (x - 1)\ (x^3 + x^2 + x - 2)$

$g(x) = x^3 - 3x^2 + 3x - 1 = (x - 1)^3$

and $h(x) = x^4 - 1 = (x - 1)\ (x + 1)\ (x^2 + 1)$

$\therefore$ HCF of $[f(x), g(x), h(x)] = (x - 1)$

Hence, option A is correct.

3). Given that, $(x - 6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$, i.e., $(x - 6)$ is a factor of both expression,

Let $f(x) = x^2 - 2x - 24$ and $g(x) = x^2 - kx - 6$

Now, $f(x) = g(x)$ at $(x = 6)$

$\Rightarrow (6)^2 - 2(6) - 24 = (6)^2 - k(6) - 6$

$\Rightarrow 36 - 12 - 24 = 36 - 6k - 6$

$\Rightarrow 0 = 30 - 6k \Rightarrow 6k = 30 \Rightarrow k = 5$.

Hence, option B is correct.

4). Let $f(x) = (x^4 - y^4) = [(x^2)^2 - (y^2)^2]$

$= (x^2 - y^2)(x^2 + y^2)$

$= (x - y)(x + y)(x^2 + y^2)$

$g(x) = (x^6 - y^6) = (x^3)^2 - (y^3)^2$

$= (x^3 + y^3)(x^3 - y^3)$

$= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$

$= (x - y)(x + y)(x^2 - xy + y^2)(x^2 + xy + y^2)$

$\therefore$ HCF of $[f(x), g(x)] = (x - y)(x + y) = x^2 - y^2$

Hence, option A is correct.

5). $f(x) = x^2 + 2x - 8 = x^2 + 4x - 2x - 8$

$= x(x + 4) - 2(x + 4) = (x - 2)(x + 4)$

$g(x) = x^3 - 4x^2 + 4x = x^3 - 2x^2 - 2x^2 + 4x$

$= x^2(x - 2) - 2x(x - 2) = (x^2 - 2x)(x - 2)$

$= x(x - 2)(x - 2)$

$h(x) = x^2 + 4x = x(x + 4)$

So, LCM of $[f(x), g(x), h(x)] = x(x - 2)(x + 4)(x - 2) = x(x + 4)(x - 2)^2$

Hence, option A is correct.

6). Let $f(x) = 8(x^5 – x^3 + x) = 4 \times 2 \times x\,(x^4 – x^2 + 1)$

and $g(x) = 28\,(x^6 + 1) = 7 \times 4[(x^2)^3 + (1^2)^3]$

$= 4 \times 7\,(x^2 + 1)\,(x^4 – x^2 + 1)$

$\therefore$ HCF of $[f(x), g(x)] = 4(x^4 – x^2 + 1)$

Hence, option A is correct.

7). Let $f(x) = (x^3 – x^2 – 2x) = x(x^2 – x – 2)$

$= x\{x^2 – 2x + x – 2\}$

$= x\{x(x – 2) + 1\,(x – 2)\} = x(x + 1)(x – 2)$

And $g(x) = x^3 + x^2 = x^2(x + 1) = x.x\,(x + 1)$

$\therefore$ LCM of $[f(x), g(x)] = x(x + 1)\,.\,x\,.\,(x – 2) = x^2\,(x + 1)(x – 2)$

$= x^2(x^2 – x – 2) = x^4 – x^3 – 2x^2$.

Hence, option C is correct.

8). Let $f(x) = a^2b^4 + 2a^2b^2 = a^2b^2(b^2 + 2)$

And $g(x) = (ab)^7 – 4a^2b^9 = a^7b^7 – 4a^2b^9 = a^2b^2(a^5b^5 – 4b^7)$

$\therefore$ HCF of $[f(x), g(x)] = a^2b^2$

Hence, option C is correct.

9). Given that, $(x + k)$ is the HCF of $(x^2 + ax + b)$ and $(x^2 + cx + d)$, i.e., $(x + k)$ is a factor of both expression

Let $f(x) = (x^2 + ax + b)$ and $g(x) = (x^2 + cx + d)$

Now, $f(x) = g(x)$ at $(x = -k)$

$\Rightarrow (-k)^2 + a(-k) + b = (-k)^2 + c(-k) + d$

$\Rightarrow (a - c)k = (b - d)$

$\Rightarrow k = \frac{(b - d)}{(a - c)}$.

Hence, option D is correct.

10). To solve this question, we can apply a formula

First Polynomial × Second Polynomial = HCF × LCM

$$\text{Other Polynomial} = \frac{\text{HCF} \times \text{LCM}}{\text{Given Polynomial}}$$

$$= \frac{(x + y)(3x^5 + 5x^4y + 2x^3y^2 - 3x^2y^3 - 5xy^4 - 2y^5)}{(x^2 - y^2)}$$

$$= \frac{3x^5 + 5x^4y + 2x^3y^2 - 3x^2y^3 - 5xy^4 - 2y^5}{(x - y)}$$

$$= 3x^4 + 8x^3y + 10x^2y^2 + 7xy^3 + 2y^4$$

Hence, option C is correct.