Trigonometry Questions for SSC Exam.

Trigonometry Quiz 9

Directions: Kindly study the following Questions carefully and choose the right answer:

1. If cos X = m cos Y and where X and Y are not equal to each other and values of angle X and Y range from 0° to 90°, then

A. $\cot\frac{X+Y}{2}=\frac{m+1}{m-1}\tan\frac{Y-X}{2}$

B. $\tan\frac{X+Y}{2}=\frac{m+1}{m-1}\cot\frac{Y-X}{2}$

C. $\cot\frac{X+Y}{2}=\frac{m+1}{m-1}\tan\frac{X-Y}{2}$

D. $\cot\frac{X+Y}{2}=\frac{m-1}{m+1}\tan\frac{X-Y}{2}$

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2. Evaluate : ($Cot^4\Theta - Cosec^4\Theta + Cot^2\Theta + Cosec^2\Theta$)

A. 1 B. 0 C. – 1 D. 2

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3. If $7\sin^2\Theta + 3\cos^2\Theta = 4$ and $0 \le \Theta \le \frac{\pi}{2}$, then the value of tan Θ is :

A. $\sqrt{\frac{3}{7}}$ B. $\sqrt{\frac{2}{7}}$ C. $\frac{1}{\sqrt{3}}$ D. $\frac{1}{\sqrt{7}}$

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4. If $\cos\Theta + \sec\Theta = 2$,then the value of $\cos^{68}\Theta + \sec^{68}\Theta$ equal to

A. 1 B. 2 C. 3 D. 68

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5. If $x = a\cos^3\Theta$, $y = b\sin^3\Theta$, then

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = ?$$

A. 1 B. 0 C. 2 D. 4

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6. Find the value of $\frac{\cos 60°}{\sin 30°} + \frac{\cos 65°.\ \operatorname{cosec} 25°}{\tan 10°.\ \tan 30°.\ \tan 45°.\ \tan 60°.\ \tan 80°}$

A. 1 B. – 1 C. 0 D. 2

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7. The value of $\cot\frac{2\pi}{20}.\cot\frac{4\pi}{20}.\cot\frac{5\pi}{20}.\cot\frac{6\pi}{20}.\cot\frac{8\pi}{20}$ **is**

A. 2 B. 0 C. 1 D. 3

8. $\frac{2\,sin^2 67^\circ + 1 + 2\,sin^2 23^\circ}{5\,cos^2\,13^\circ - 2 + 5\,cos^2\,77^\circ}$ = ?

A. $\frac{3}{2}$ B. 3 C. $\frac{2}{3}$ D. 1

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9. Evaluate : (sin $\frac{\pi}{6}$ **+ cos** $\frac{\pi}{3}$ **– tan**^{3} $\frac{\pi}{4}$ **+ cosec**^{2} $\frac{\pi}{2}$**)**

A. 1 B. 0 C. 3 D. 5

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10. The least value of 9 cosec^2 A + 16 sin^2 A is

A. 7 B. 24 C. 25 D. 14

Correct Answers:

1	2	3	4	5	6	7	8	9	10
A	B	C	B	A	D	C	D	A	B

Explanations:

1. $\cos X = m \cos Y$

$$\frac{\cos X}{\cos Y} = \frac{m}{1}$$

By componendo and dividendo,

$$\frac{\cos X + \cos Y}{\cos X - \cos Y} = \frac{m + 1}{m - 1}$$

$$\frac{2 \cos \left(\frac{X + Y}{2}\right). \cos \left(\frac{X - Y}{2}\right)}{-2 \operatorname{Sin}\left(\frac{X + Y}{2}\right). \sin \left(\frac{X - Y}{2}\right)} = \frac{m + 1}{m - 1}$$

$$\frac{\cos \left(\frac{X + Y}{2}\right)}{\sin \left(\frac{X + Y}{2}\right)} \cdot \frac{\cos \left(\frac{Y - X}{2}\right)}{\sin \left(\frac{Y - X}{2}\right)} = \frac{m + 1}{m - 1}$$

$\because \cos(-X) = \cos X$ whereas $\sin(-X) = -\sin X$

$$\cot\frac{X + Y}{2}. \cot\frac{Y - X}{2} = \frac{m + 1}{m - 1}$$

$$\cot\frac{X + Y}{2} = \frac{m + 1}{m - 1}. \tan\frac{Y - X}{2}$$

$$\left[\because \cot \Theta = \frac{1}{\tan \Theta}\right]$$

Hence, option A is correct.

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2. Given, $(\operatorname{Cot}^4 \Theta - \operatorname{Cosec}^4 \Theta + \operatorname{Cot}^2 \Theta + \operatorname{Cosec}^2 \Theta)$

$= (\operatorname{Cot}^4 \Theta + \operatorname{Cot}^2 \Theta) - (\operatorname{Cosec}^4 \Theta - \operatorname{Cosec}^2 \Theta)$

$= \operatorname{Cot}^2 \Theta (\operatorname{Cot}^2 \Theta + 1) - \operatorname{Cosec}^2 \Theta (\operatorname{Cosec}^2 \Theta - 1)$

$= \operatorname{Cot}^2 \Theta . \operatorname{Cosec}^2 \Theta - \operatorname{Cosec}^2 \Theta . \operatorname{Cot}^2 \Theta$ $\quad [\because (\operatorname{Cot}^2 \Theta + 1) = \operatorname{Cosec}^2 \Theta \ \& \ (\operatorname{Cosec}^2 \Theta - 1) = \operatorname{Cot}^2 \Theta]$

$= 0$

Hence, option B is correct.

3. $7\sin^2\Theta + 3\cos^2 = 4$

$\Rightarrow 7\sin^2\Theta + 3(1 - \sin^2\Theta) = 4 \Rightarrow \sin^2\Theta = \frac{1}{4}$ .so, $\sin\Theta = \frac{1}{2}$

$\Rightarrow 7\sin^2\Theta + 3(1 - \sin^2\Theta) = 4 \Rightarrow \sin^2\Theta = \frac{1}{4}$ .so, $\sin\Theta = \frac{1}{2}$

$\therefore \cos\Theta = \sqrt{1-\sin^2\Theta} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

$\therefore \tan\Theta = \frac{\sin\Theta}{\cos\Theta} = \left(\frac{1}{2} \times \frac{2}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}}$

Hence, option C is correct.

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4. $\cos\Theta + \sec\Theta = 2$

or, $\cos\Theta + \frac{1}{\cos\Theta} = 2$

or, $\cos^2\Theta + 1 = 2\cos\Theta$

or, $\cos^2\Theta - 2\cos\Theta + 1 = 0$

or, $(\cos\Theta - 1)^2 = 0$

$\therefore \cos\Theta = 1$

$\therefore \sec\Theta = \frac{1}{\cos\Theta} = 1$

Now, $\cos^{68}\Theta + \sec^{68}\Theta = 1 + 1 = 2$

Hence, option B is correct.

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5. $x = a\cos^3\Theta$

$\therefore \frac{x}{a} = \cos^3\Theta$

and $y = b\sin^3\Theta$

$\therefore \frac{y}{b} = \sin^3\Theta$

Now, $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3}$

$= (\cos^3\Theta)^{2/3} + (\sin^3\Theta)^{2/3}$

$= \cos^2\Theta + \sin^2\Theta = 1$ [$\because \cos^2\Theta + \sin^2\Theta = 1$

Hence, option A is correct.

6. **Given Expression:**

$$\frac{\cos 60°}{\sin 30°} + \frac{\cos 65° \,.\, \text{coses } 25°}{\tan 10° \,.\, \tan 30° \,.\, \tan 45° \,.\, \tan 60° \,.\, \tan 80°}$$

$$\frac{\cos (90° - 30°)}{\sin 30°} + \frac{\cos 65° \,.\, \text{cosec } (90° - 65°)}{\tan 10° \,.\, \tan 30° \,.\, \tan 45° \,.\, \tan (90° - 30°) \,.\, \tan (90° - 10°)}$$

$$\frac{\sin 30°}{\sin 30°} + \frac{\cos 65° \,.\, \sec 65°}{\tan 10° \,.\, \tan 30° \,.\, \tan 45° \,.\, \cot 30° \,.\, \cot 10°}$$

$$= 1 + \frac{\cos 65° . \frac{1}{\cos 65°}}{\tan 10° \,.\, \tan 30° \,.\, \tan 45° \,.\, \cot 30° \,.\, \cot 10°}$$

$$= 1 + \frac{1}{1} = 2$$

Hence, option D is correct.

7.

Given expression: $\cot \frac{2\pi}{20} \,.\, \cot \frac{4\pi}{20} \,.\, \cot \frac{5\pi}{20} \,.\, \cot \frac{6\pi}{20} \,.\, \cot \frac{8\pi}{20}$

$$= \cot \frac{\pi}{10} \,.\, \cot \frac{\pi}{5} \,.\, \cot \frac{\pi}{4} \,.\, \cot \frac{3\pi}{10} \,.\, \cot \frac{2\pi}{5}$$

$$= \cot \frac{180°}{10} \,.\, \cot \frac{180°}{5} \,.\, \cot \frac{180°}{4} \,.\, \cot \frac{3 \times 180°}{10} \,.\, \cot \frac{2 \times 180°}{5}$$

$$= \cot 18° \,.\, \cot 36° \,.\, \cot 45° \,.\, \cot 54° \,.\, \cot 72°$$

[we know that (18, 72) and (36, 54) are complementary angles, so $\cot (90° - \Theta) = \tan\Theta$ or, $\cot 54° = \tan (90° - 54) = \tan 36°$]

$$= \frac{1}{\tan 18°} \times \frac{1}{\tan 36°} \times \cot 45° \,.\, \tan 36° . \tan 18°$$

$\Rightarrow \cot 45° = 1$

Hence, option C is correct.

8.

Given expression: $\frac{2\sin^2 67° + 1 + 2\sin^2 23°}{5\cos^2 13° - 2 + 5\cos^2 77°}$

$\Rightarrow \frac{2\sin^2 67° + 2\sin^2 23° + 1}{5\cos^2 13° + 5\cos^2 77° - 2}$

$\Rightarrow \frac{2(\sin^2 67° + \sin^2 23°) + 1}{5(\cos^2 13° + \cos^2 77°) - 2}$

$\Rightarrow \frac{2[\sin^2 67° + \sin^2 (90° - 67°)] + 1}{5[\cos^2 13° + \cos^2 (90° - 13°)] - 2}$

$\Rightarrow \frac{2(\sin^2 67° + \cos^2 67°) + 1}{5(\cos^2 13° + \sin^2 13°) - 2}$

$\Rightarrow \frac{2 \times 1 + 1}{5 \times 1 - 2} = \frac{3}{3} = 1$

Hence, option D is correct.

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9. We know that:

$\sin \frac{\pi}{6} = \sin 30° = \frac{1}{2}$, $\cos \frac{\pi}{3} = \cos 60° = \frac{1}{2}$;

$\tan \frac{\pi}{4} = \tan 45° = 1$

and $\operatorname{cosec} \frac{\pi}{2} = \frac{1}{\sin \frac{\pi}{2}} = \frac{1}{\sin 90°} = 1$

$\therefore \sin \frac{\pi}{6} + \cos \frac{\pi}{3} - \tan^3 \frac{\pi}{4} + \operatorname{cosec}^2 \frac{\pi}{2}$

$= \left(\frac{1}{2} + \frac{1}{2} - 1^3 + 1^2\right) = 1$

Hence, option A is correct.

10. $9\ \text{cosec}^2 A + 16 \sin^2 A$

$= \frac{9}{\sin^2 A} + 16 \sin^2 A$

$= \left(\frac{3}{\sin A}\right)^2 + (4 \sin A)^2$

[$\because\ a^2 + b^2 = (a - b)^2 + 2ab$]

Let $a = \frac{3}{\sin A}$, $b = 4 \sin A$

$= \left(\frac{3}{\sin A} - 4 \sin A\right)^2 + 2 \times \frac{3}{\sin A} \times 4 \sin A$

$= \left(\frac{3 - 4 \sin^2 A}{\sin A}\right)^2 + 24$

For the least value of $\left(\frac{3 - 4\sin^2 A}{\sin A}\right)$ should be 0.

$\therefore$ The least value will be 24.

Hence, option B is correct.